Graphical Abstract

Three-dimensional Deep Shape Optimization with a Limited Dataset

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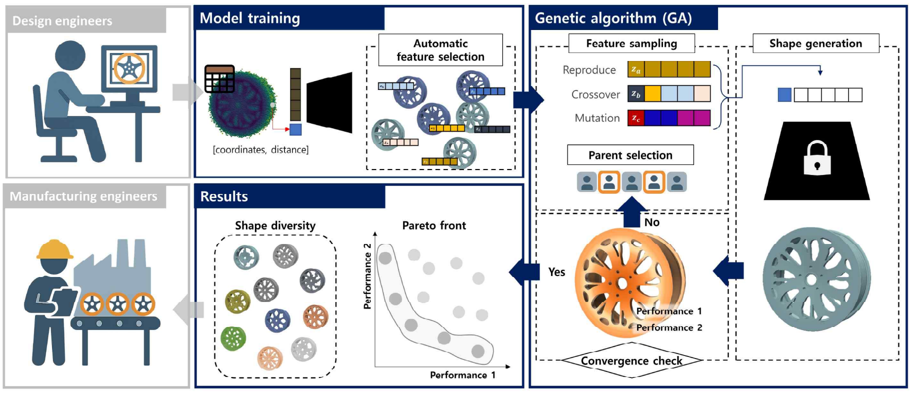

Highlights

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- O ercomes e plicit parameterization limits sing data-dri en shape e traction
- Optimizes 3D shapes effectively with a limited dataset approach.
- Achie es di erse design o tcomes in m lti-ob ecti e optimization
- alidates the ad anced per ormance o the ramewor thro gh m ltiple e periments

Three-dimensional Deep Shape Optimization with a Limited Dataset

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Abstract

Generati e models ha e attracted considerable attention or their abilit to prod ce no el shapes Howe er their application in mechanical design remains constrained d e to the limited size and ariabilit o a ailable datasets This study proposes a deep learning-based optimization framework specifically tailored or shape optimization with limited datasets le eraging positional encoding and a Lipschitz reg larization term to rob stl learn geometric characteristics and maintain a meaning l latent space Thro gh e tensi e e periments the proposed approach demonstrates rob stness generalizability and effectiveness in addressing typical limitations of conventional optimization frameworks. The validity of the methodology is confirmed through m lti-ob ecti e shape optimization e periments cond cted on di erse three-dimensional datasets incl ding wheels and cars highlighting the model s ersatilit in prod cing practical and high- alit design o tcomes e en nder data-constrained conditions

e words Signed Distance Function, Artificial Intelligence, Limited Dataset mplicit e ral epresentation Shape Optimization

ntrod ction

Artificial intelligence (AI) has shown significant achievements in various domains, including regression, classification and segmentation [1, 2]. Building pon these ad ancements research in mechanical design s ch as comp ter-

* orresponding a thor

aided design AD and comp ter-aided engineering has increasingl ti-lized A to o ercome the limitations o traditional methodologies and pro ide new insights Deep generati e models DG s ch as the ariational a toencoder A the generati e ad ersarial networ (GAN) and the diffusion model, have garnered increasing attention due to their abilit to learn rom data and create new data that were not pre io sl accessible [12, 13, 14]. This characteristic is a significant solution to the persistent data scarcit problem in the mechanical domain nli e con entional generati e design approaches DG g arantees data di ersit acilitating the generation o design s ggestions that incorporate engineering nctionalit

Traditional parametric shape optimization produces different optimal results depending on how feature selection is performed. This difference becomes more prono nced or comple shapes with highl dependent parameters n contrast A models can a tomaticall anal ze the eat res o data thro gh learning and compress data into a latent space rom an optimal design perspecti e the latent space can be sed as design ariables pro iding a tomatic eat re selection nctionalit

One of the significant issues in employing DGM for mechanical design lies in the need or aried data within the same categor echanical parts which are ndamentall mass-prod ced do not ha e ab ndant data within the same design domain and e isting data show onl minimal ariation These intrinsic data characteristics create significant challenges for AI when attempting to identi s btle distinctions within a single categor as it tends to oc s on the o erall orm which can lead to iss es li e mode collapse

This st d aims to propose a three-dimensional D deep learning model tailored or optimal design applications The proposed model a tomaticall selects eat res rom generati e A training data while preser ing shape di ersit rthermore the proposed model demonstrated rob st per ormance on limited datasets, addressing a bottleneck previously identified in optimal design approaches This research introd ces a ersatile shape optimization ramewor sing A models This st d in estigates shape optimization across distinct datasets an endea or that con entional parametric techni es and e isting DG -based approaches ha e et to achie e n addition this ramewor is applied to shape optimization problems demonstrating its universality through optimizations performed via computational fluid dynamics (CFD) and finite element method (FEM) simulations.

This paper is str ct red as ollows Section introd ces pre io s research on shape optimization topolog optimization and shape optimization sing DG and disc sses data representation and deep learning architect re in the conte t o D data Section pro ides an o er iew o the entire ramewor o this st d rom data collection to shape optimization Section e amines the originalit and alidit o the methodolog thro gh three shape optimization e periments inall Section presents concl sions and directions or t re wor

elated or s

Shape Optimization ethod

arametric Shape Optimization

Data generated thro gh AD programs consists o combinations o ario s geometric ariables which can be tilized as design parameters in the optimization process arametric shape optimization is a design method in which design parameters, within a predefined range, satisfy constraints based on a defined CAD model while simultaneously identifying the optimal shape to minimize the ob ecti e nction This method is widel sed in various engineering fields, including automotive, aerospace, electromagnetic and aco stic

arametric shape optimization has ad anced with s rrogate modeling or predicting engineering performance because it allows for defining data within a specified range via design parameters. In particular, substantial progress has been made in areas where the data can be easily defined by simple curves or encaps lated b n merical characteristics perimental res lts indicate that the e tensi e design space and intricate parameter dependencies enable he ristic optimization methods s ch as the on-dominated Sorting Genetic Algorithm SGA- to o tper orm gradient-based approaches

or non-parametric data s ch as D meshes parameterization can be per ormed sing techni es li e pol c be mapping or ree- orm de ormation These methods can be implemented relati el easil and trans orm comple and detailed shapes rthermore when constr cting s rrogate models mesh data can be represented as a graph and processed with graph ne ral networ s enabling the sol tion o more generalized problems Howe er a limitation o these methods is that depending on the resolution of the defined parameters, capturing fine details becomes challenging and the n mber o parameters increases

Topolog Optimization-based Generati e Design

Topology optimization is the most effective method for generating shapes that satisfy prescribed boundary conditions and meet specific objective functions This methodolog has ndergone n mero s ad- ancements since introd cing the initial solid isotropic material with penalization approach in the academic literat re

It is the most widely utilized method in the field of generative design because it can generate new data based on constraints defined by designers n generati e design topolog optimization e plores new shapes throughout the design process, from problem definition to detailed design This approach streamlines the process b n mericall e al ating and generating optimal designs prod cing better o tcomes than traditional stage-b -stage methods

Generati e models that le erage topolog optimization e cel in data generation nder m ltiple bo ndar conditions allowing the prod ction o ario s designs Howe er a notable drawbac is that the tend to ield man similar data instances To o ercome these limitations recent st dies ha e increasingl oc sed on combining topolog optimization with deep generati e models or e ample Oh et al sed topolog optimization to create seed data to train a deep generati e model The then trained a GA model to generate diverse wheel designs with various spoke configurations. rthermore there are emerging cases that incorporate rein orcement learning to address design di ersit and m lti-ob ecti e re irements

Deep Generati e odel-based Shape Optimization

ecentl research on deep generati e models has been ad ancing signi icantl tensi e wor is being carried o t on creati e generati e models such as GANs, VAEs and diffusion models, which are capable of generating new images and shapes Deep generati e models ha e e ol ed according to the t pe o data in ol ed or instance the are partic larl effective for processing non-parametric data such as images or 3D meshes and extracting low-dimensional features. In the field of optimal design, research is being cond cted on generating and optimizing shapes sing deep generati e models

Optimization studies utilizing DGMs have seen significant advancements in domains that are well-s ited or image representation s ch as meta str ct res or air oils which are relati el eas to parameterize In particular, new architectures have been developed to better define airfoils, typically represented by Bézier curves, to improve the feasibility of the generated data

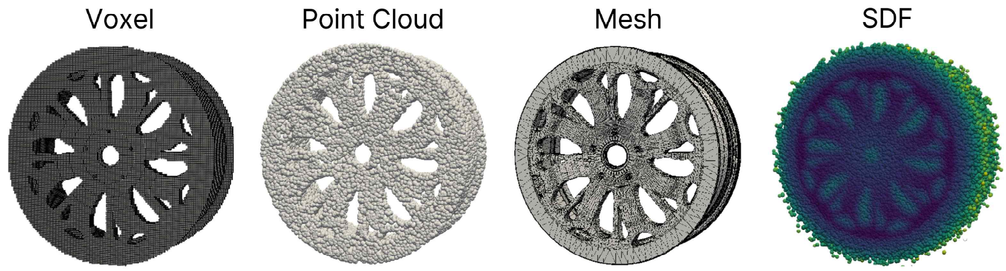
or D data architect res based on implicit ne ral representations as introd ced b ar et al ha e been widel adopted These models are highl ersatile ser ing not onl as generati e models b t also as s r-rogate models. Their ability to perform auto-differentiation makes sensi-ti it anal sis straight orward ma ing them well-s ited or capt ring local ariations This approach has been partic larl se l or ehicle aerod -namic optimization problems which t picall re ire high comp tational reso rces

O erall data-dri en deep generati e models disting ish themsel es rom topolog optimization-based generati e design in that the can e plore more realistic data distributions and discover unique configurations. However, they re ire large amo nts o data

	Topolog Optimization	arametric Shape Optimization	DG -based Shape Optimization	roposed ethod
arameterization	A tomatic	an al	A tomatic	A tomatic
nterpolation	n easible	n easible	ossible	ossible
on ergent possibilit	ad	Good	Good	Good
Training data	one	one	Large	Small minim m

Table omparison with other optimization methods

n s mmar Section is concisel represented b Table Altho gh DG -based shape optimization s ccess ll addresses the limitations o tra-ditional topolog optimization and parametric shape optimization meth-ods, it still suffers from a critical drawback: requiring an extensive training dataset This st d addresses this iss e b proposing a rob st model that deli ers strong per ormance e en with limited datasets ma ing it partic -larl s itable or shape optimization applications Detailed descriptions and e planations are pro ided in Section



ig re o r common D data representations

Deep Learning or D Data

D Data epresentation

Deep learning with D data representation can ar depending on the specific problem being addressed. Representation for data-driven 3D deep learning can be broadl categorized into o el point clo d and mesh representation as shown in ig

irst o el is a representation method that discretizes D ol me into a grid This is ad antageo s beca se it is similar to the representation o images ma ing it relati el eas to appl to traditional ne ral networ s s ch as con ol tional ne ral networ s Howe er sing the o el approach to represent D shapes can lead to memor iss es rthermore representing a D shape as a o el can res lt in relati el nnecessar areas leading to higher computational costs. Voxel representation is heavily influenced by resol tion which can ma e the shapes relati el less detailed and bl nt

oint clo d is a prominent representation method among D representations, particularly in applications such as classification, retrieval and segmentation t represents a D s r ace sing D points allowing or detailed and global shape representations. However, it has a significant drawback in that it cannot capt re connecti it or topological in ormation between indi id al points and cannot generate watertight meshes

inall mesh is a representation method that connects ertices and edges between them to represent the s r ace o a D shape This method allows for the representation of detailed shapes and is memory-efficient, making it a pop lar choice in generati e models Howe er changing the topolog can be challenging and sel -intersection iss es can arise when de orming the mesh As a res lt it is not a s itable representation or mechanical design with ario s topological changes

At present these representation methods are not s itable or learning D shapes which is wh there is growing interest in representing data implicitly. Implicit functions involve constructing a continuous volumetric field and embedding shapes as iso-s r aces with the signed distance nction SD being a prominent e ample This can be e pressed in terms o the coordinates in space as the signed distance s to the s r ace as shown in The SD represents the shortest distance rom a point to the shape s s r ace while indicating whether the point is inside or o tside the shape based on its sign. Specifically, for points inside the shape, the distance is negative $(-$ or points o tside it is positi e and it is zero on the s r ace

$$SD \quad s \quad s$$

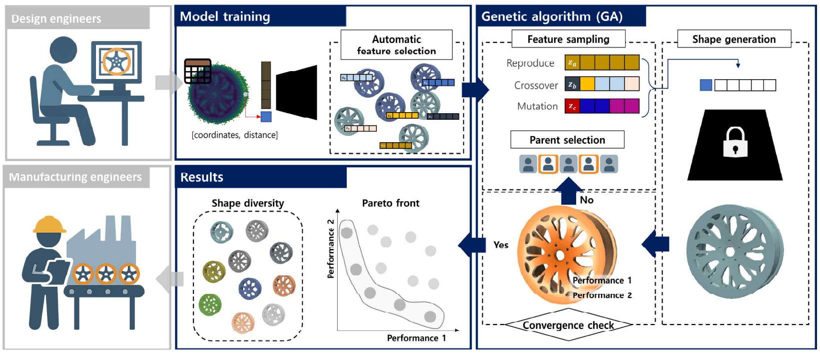
The s r ace o a shape represented b SD can be e pressed as the iso-s r ace o SD $\cdot$ and this can be con erted into a discretized s r ace sing algorithms s ch as marching c bes

mplicit e ral epresentation

mplicit representations s ch as SD enable implicit learning where coordinates ser e as inp ts to an A model and predict the distance al e corresponding to those coordinates Since this t pe o learning does not rel on discretized data th s it theoreticall enables e porting D shapes with infinite resolution and handling topological changes easily. Implicit learning can densel capt re important parts o act al shapes acilitating the generation of feasible and novel shapes. Starting with DeepSDF, the first to propose representing shapes as SD s and learning them implicitl se - eral models ha e been introd ced or shape manip lation These incl de D alSD which adds a coarse networ or shape manip lation and A-SD which disentangles the latent space or artic lated ob ects All o these models eat re an a to decoder AD str ct re witho t an encoder nli e man generati e model st dies that red ce data dimension within a bottlenec str ct re these models do not ha e a separate encoder or data compression nstead the AD inp ts both coordinates and latent code into the model and bac propagation optimizes latent code within the model

ncoder decoder D models that tilize encoders s ch as oint et to compress data e ist These str ct res create latent codes thro gh an encoder which has been sed to handle implicit data The wor b eng et al proposed a methodolog or e tracting both global and local eat res rom point clo ds enabling the reconstr ction o large D scenes The paper introd ced the idea o encoding point clo ds into two-dimensional planes or D ol mes trans orming nstr ct red point clo d data into str ct red latent space. This methodology allows for assigning different latent codes to indi id al points enabling learning more detailed eat res in shapes The emplo ed a - et or decoding sing s ip connections in the - et architect re the co ld sim ltaneo sl learn both local and global eat res significantly improving the accuracy of occupancy probability for each point. The deri ed latent ector and coordinates are then ed into an SD networ to predict the distance

ethodolog



ig re O er iew o the proposed optimization

The o erall ramewor o this st d is ill strated in ig which is all s mmarizes each step o the process rom data collection and preprocessing to model training eat re sampling and shape generation

Data ollection

n the initial phase a limited dataset comprising -wheel and -car data rom both topolog optimization and parametric design methods was carefully selected to demonstrate that optimization can be effective even with limited data reprocessing then ens red data alit thro gh chec s for mesh flips and watertightness [60], followed by centering, normalization and sampling s r ace-ad acent points with corresponding SD e traction

odel Training A tomatic eat re Selection

reprocessed data consisting o spatial coordinates and corresponding signed distances enabled the model to learn their relationships D ring training latent codes were optimized alongside the model s weights to enhance learning. To robustly capture fine details within a limited dataset, the implicit ne ral representation was impro ed b incorporating positional encoding or high- re enc in ormation and emplo ing Lipschitz reg larization to b ild a meaning l latent space pon training completion the learned decoder s ccess ll reconstr cted non-parametric data and generated a latent space that effectively vectorized shape information for the extraction of geometric eat res

eat re Sampling

Operator	sed
arent selection	To rnament selection
rosso er	Sim lated binar crosso er
tation	ol nomial m tation

Table T pes o operators sed

ith the training phase complete the decoder was rozen and eat re sampling was cond cted rom the contin o s latent space A genetic algorithm GA was emplo ed as the optimization method wherein a pop lation is e ol ed o er s ccessi e generations thro gh e operators parent selection sim lated binar crosso er and pol nomial m tation to search for optimal solutions. Table 2 summarizes the specific operators used. In GA, individuals with higher fitness are selected as parents, combined via crosso er and then ndergo m tation to maintain di ersit and red ce the ris o con erging to s boptimal sol tions This approach is well-s ited or comple optimization challenges enabling A -based shape optimization b refining the latent code fed into the decoder. In particular, the NSGA-2 algorithm emplo s crowding distance calc lations to e plore a broader range o sol tions and nco er di erse shape designs

Shape Generation Sim lation

sing latent codes prod ced b the optimization algorithm the decoder generated new shapes b prod cing a SD This SD was then con erted into a D triang lar mesh sing the marching c bes algorithm

The per ormance o the generated shapes was e al ated ia sim lations sim lations were cond cted sing Altair nspire while D sim lations were per ormed sing Open OA Gi en the m lti-ob ecti e nat re of the optimization, two performance metrics with inherent trade-offs were selected: stiffness and mass for the wheel data and the drag coefficient ($_D$ and lift coefficient ($_L$ or the ehicle data

on ergence hec

on ergence was determined b e al ating per ormance metrics across a predefined maximum number of generations. If convergence wasn't achieved, the process ret rned to eat re selection to create a new pop lation or the ne t generation con ergence was reached the algorithm was terminated

etwor Architect re

echanical design prod cts ha e bo ndar conditions s ch as load and fixed conditions, resulting in minimal differences among the data. Therefore, it is important or generati e models o mechanical design domains to learn the features of each dataset without significant differences between them.

Training on s ch a dataset with a standard m lti-la er perceptron L ma be problematic or two reasons irst an L might ail to capt re the ni e eat res o each data sample which is critical gi en that man mechanical components exhibit significant performance variations due to subtle differences. Second, the limited number of available samples within the same class poses a challenge Hence the A model m st acc ratel learn the eat res o the dataset e en with sparse data while de eloping a meaning l latent space ositional encoding and Lipschitz reg larization terms ha e been introd ced to address these iss es

ositional encoding as o tlined b ildenhall et al and Sitzmann et al is a power l techni e that pro ects inp t coordinates into a high-dimensional o rier space thro gh sin soidal trans ormations as represented in

$$\gamma \, \mathrm{p} \quad \sin \quad \pi p \;\; \cos \quad \pi p \qquad \sin \;{}^{\mathrm{L}-}\; \pi p \;\; \cos \;{}^{\mathrm{L}-}\; \pi p$$

p represents the inp t coordinates and L specifies the frequency levels in the encoding This method allows the model to capt re high- re enc information effectively before passing it to MLP.

encoding position in ormation across a range o re encies the model is able to learn ario s spatial eat res and disting ish s btle ariations within the defined space. Such granularity is advantageous for learning detailed SDFs, enabling the model to capture fine details in spatially complex data

Furthermore, these studies have confirmed that projecting inputs into a higher-dimensional space sing high- re enc nctions s ch as o rier transformations, is highly effective for encoding complex information. This approach enhances model e pressi eness and precision in capt ring details

As presented b Li et al the Lipschitz reg larization term smooths the latent space b constraining the magnit de o each la er s weight matri This reg larization is applied to each la er s weight matri within the L as represented b the la er e ation eL $_i$ b$_i$

$$\text{eL} \quad {}_{i} \quad b_{i} \quad {}_{i} \quad \text{normalization} \quad {}_{i} \ \ln \quad e^{\,i}$$

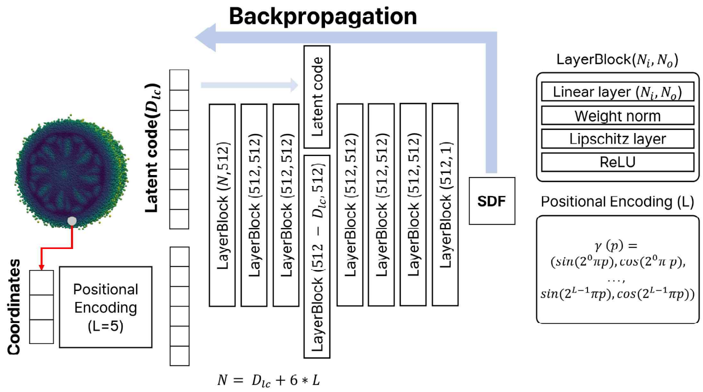
or all l la ers o the decoder the Lipschitz la er normalizes the weight matri $_i$ o each i-th linear la er sing trainable Lipschitz bo nds $_i$ as shown in

$$\text{loss}_{\text{Lipschitz}} \quad \overset{i}{\underset{l}{}} \ \ln \quad e^{\,i}$$

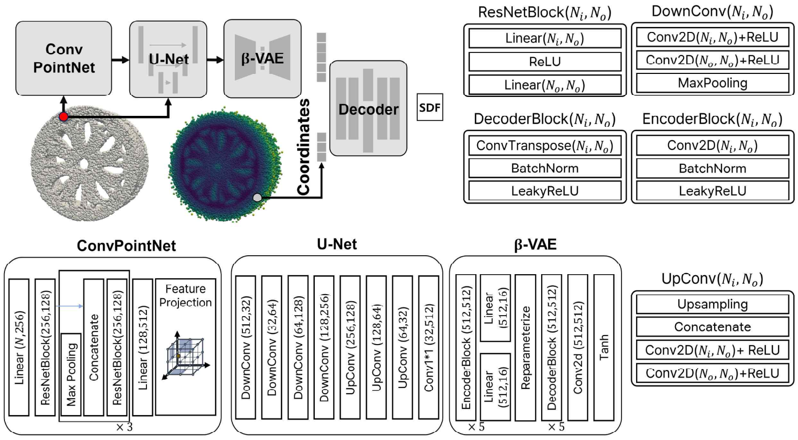
The o erall Lipschitz bo nd or the networ is determined b the prod ct o the indi id al Lipschitz constants $_i$ assigned to each la er As shown in a loss nction can en orce this constraint stabilizing the model b pre enting weight al es rom e ceeding a set threshold at each la er This reg larization limits the model s response to inp t data red cing e cessi e sensiti it and enhancing o erall model stabilit

This method is particularly beneficial for synthesis tasks with limited data, as it confines the model's output to a compact latent space, allowing a comprehensive representation of training data and effective pattern learning ithin the broader ramewor o limited-data optimization Lipschitz reg larization pla s a pi otal role in ostering rob st learning and ens ring reliable per ormance

Two architect res were e al ated an AD str ct re witho t an encoder and an D str ct re that integrates a oint et-based encoder $_\phi$ as shown in ig This encoder inp ts point clo d data π_{pcd} and e tracts latent eat res oth models are trained sing a latent space reg larization term and a tr ncated L loss to meas re discrepancies between predicted and true SDF values. The truncated L1 loss, a modification of standard L1 loss, tr ncates both positi e and negati e o tliers to limit e treme de iations while preserving sensitivity within a specified range (δ as ill strated in n e periments δ was set to and β was set to



a A to decoder



b ncoder decoder

ig re odel architect res

$$
\mathrm{L}_{\mathrm{clip}} \;\; \mathrm{d}_{\mathrm{pred}} \;\; \mathrm{d}_{\mathrm{gt}} \qquad
\begin{array}{ll}
\mathrm{ma} \;\; \mathrm{d}_{\mathrm{pred}} \; -\delta \qquad \delta & d_{\mathrm{gt}} \quad -\delta, \\
\mathrm{d}_{\mathrm{pred}} - \mathrm{d}_{\mathrm{gt}}| & -\delta < d_{\mathrm{gt}} < \delta, \\
\delta - \min \; \mathrm{d}_{\mathrm{pred}}, \delta & \mathrm{d}_{\mathrm{gt}} > \delta.
\end{array}
$$

$$\mathrm{L}_{\mathrm{AD}} \quad \mathrm{L}_{\mathrm{clip}} \quad \theta \quad \mathrm{z} \quad \mathrm{d}_{\mathrm{gt}} \quad \mathrm{z} \quad \mathrm{w}_{\mathrm{AD}}\,\mathrm{loss}_{\mathrm{Lipschitz}}$$

$$\mathrm{L} \quad \mathrm{D} \quad \mathrm{L}_{\mathrm{clip}} \quad \theta \quad \mathrm{z} \quad \mathrm{d}_{\mathrm{gt}} \quad \beta \; \mathrm{D} \quad \mathrm{L} \quad \phi \;\; \mathrm{z} \;\; \pi_{\mathrm{pcd}} \quad \theta \;\; \mathrm{Z} \quad \mathrm{w} \quad \mathrm{D}\,\mathrm{loss}_{\mathrm{Lipschitz}}$$

The overall loss function comprises multiple components, as defined in s and

Let $_\theta$ denote the A model implicitl learning SD Here z_i represents the latent ector embedding each shape while and d_{gt} indicate the spatial coordinates and the corresponding gro nd tr th distance The β- A was trained b comp ting the L di ergence D $_L$ between the learned latent distrib tion and the prior To align the scale with L_{clip} and D $_L$ weights o w_{AD} × $^-$ or the AD model and w $_D$ × $^-$ or the D model were applied to the Lipschitz loss term

An AD str ct re witho t an encoder was adopted the rationale behind this design choice is detailed below

econstr ction per ormance

econstr ction per ormance was compared or the AD and D models sing cham er distance D minim m matching distance D and co erage O metrics as introd ced b ang et al These metrics serve specific purposes. The formulas for CD, MMD and COV are as s and respecti el Let $_g$ and $_r$ denote the sets o generated and re erence point clo ds respecti el with $_r$ $_g$ The distance D · · between two point clo ds is comp ted sing either D or earth mo er s distance D meas res similarit D meas res alit and O meas res di ersit O ser es as an indicator or mode collapse

$$D \quad {}_g \quad {}_r \qquad \sum_{g \quad g} \min_{r \quad r} \quad g - r \qquad \sum_{r \quad r} \min_{g \quad g} \quad r - g$$

$$D \quad {}_g \quad {}_r \qquad \frac{}{r} \sum_{r \quad r} \min_{g \quad g} D \quad g \quad r$$

$$O \quad {}_g \quad {}_r \qquad \frac{\mathrm{argmin}_{r \quad r} D \quad g \quad r \quad g \qquad {}_g g}{r}$$

To e al ate both models e periments were per ormed on car shapes rom Shape et and car wheels generated ia deep generati e models and parametric design econstr ctions were obtained sing the trained models with sampled points sed to comp te metrics As shown in Table the AD model consistentl o tper ormed the D model or both car and wheel shapes with the D mean indicating that the AD handles subtle variations effectively. Although the D mean a ored the AD the D median was lower or the D s ggesting that mode collapse impacted the D s per ormance partic larl in smaller latent spaces oreo er as shown in ig increasing the latent dimension in the AD led to difficulties in reconstr cting thin str ct res s ch as wheel rims while a smaller latent dimension allowed the model to capture fine features more comprehensi el preser ing a di erse and meaning l latent space

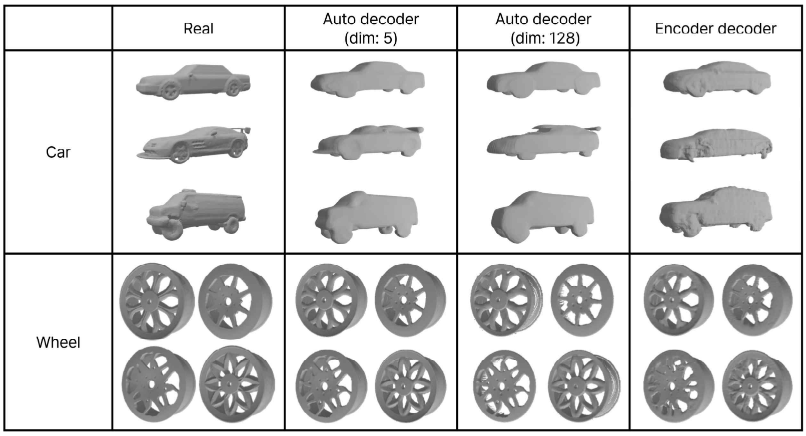
Data	Str ct re	Latent dimension	Time	D mean	D median	D	O
ar	A to decoder		h				
	ncoder decoder		d				
heel	A to decoder		h				
	ncoder decoder		d				

Table Training res lts

elati el short training time

The D str ct re incorporates an encoder res lting in to times more parameters than the AD str ct re which contains parameters oth the AD and D models were trained on paired data points or SD learning and the D model additionall incorporated point-clo d samples into the training process

The slower training speed o the D model is primaril d e to its comple processing rather than st a larger parameter co nt omprising a β- A oint et and - et or global eat re e traction the D model per orms additional operations li e pro ecting point clo ds onto two dimensions increasing processing time e ond its comp tational b rden the model per orms additional internal tas s s ch as pro ecting point clo ds into two-dimensional planes These operations consume considerable processing time, significantly challenging the overall training speed. Specifically, training the ED model on 30 samples re- ired abo t da s on an idia A G o er epochs which is a significant drawback when rapid training is required. In contrast, the simplicit o the AD str ct re red ces comp tational o erhead and training time dramaticall it re ires onl abo t ho rs o er epochs, highlighting its efficiency for rapid deployment.



ig re econstr cted D shapes or car and wheel datasets perimen ts

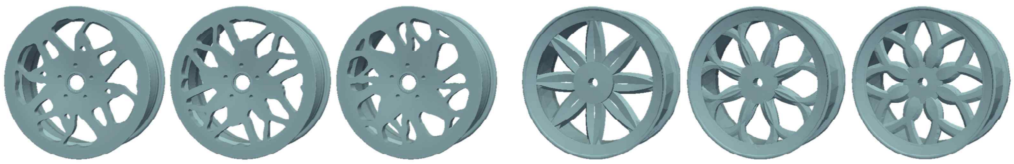
Dataset	Ob ecti e	periment	o data	Latent Dimension	se o e ral etwor
heel	max: stiffness min mass	Data S nthesis			S nthesize the other t pes o data
		tremel Limited Data			plore shapes between data
ar	min D L	D Anal sis			Generate good- alit shapes or engineering anal sis
		tremel Limited Data			plore shapes between data

Table periment descriptions

n this section e periments were cond cted to demonstrate the no elt and ersatilit o the proposed methodolog artic larl this st d tilizes data-dri en methods to generate shapes with ario s eat res based on the training data lti-ob ecti e optimization was per ormed to le erage the abilit to e plore di erse shapes in the proposed approach The proposed methodolog s ersatilit was demonstrated thro gh e periments cond cted in the conte t o shape optimization across distinct data t pes D-based shape optimization and optimization between two shapes The e periment concerning the optimization between two shapes was cond cted sing car data and wheel data A brie description o the e periments is pro ided in Table periments cond cted in Section and were per ormed with a pop lation size o indi id als per generation iterating thro gh a total o generations Similarl the e perimental settings described in Section emplo ed a pop lation size o indi id als per generation o er generations

Shape Optimization across Distinct Data T pes

Traditional parametric shape optimization represents arbitrar shapes using a single set of parameters, potentially sacrificing some intrinsic degrees o reedom Similarl red cing the ll dimensionalit o D inp t shapes to a compressed latent space ector ma res lt in some loss o in ormation However, the learned latent space is designed to capture the most significant shape ariations which means it is more li el to preser e critical degrees



a Generati e design data b arametric design data

Figure 5: Different wheel design variations

o reedom across a broader range o shapes than con entional parametric representation techni es

As seen in ig e en when the same wheel is designed data can be defined in a variety of ways. Defining various datasets as a single parametric set is impossible As a res lt no research on optimizing them has been cond cted The methodolog o this st d is based on data rather than parametric modeling which is wh it is not hea il dependent on the data generation process To demonstrate this condition e periments were cond cted sing a dataset comprising engineer-designed wheels and wheels generated b a deep generati e model The selection process in ol ed identi ing wheel data with a spo e pattern composed o m ltiples o allowing or a s mmetrical shape achie ed thro gh a -degree rotation considering the significant characteristic of cyclic symmetry in wheel data. In this study, stiffness is obtained by analyzing the natural frequencies of the wheels thro gh modal anal sis sing the ree- ree method The ree- ree method in modal anal sis e amines the nat ral re encies and mode shapes o a str ct re in an nconstrained state meaning the ob ect has no fixed boundaries or supports. This is done by solving the eigenvalue problem deri ed rom the s stem s e ations o motion which are based on its mass and stiffness properties. The natural frequencies correspond to the square roots o the eigen al es indicating the rates at which the str ct re ibrates nat rall This approach allows or an acc rate assessment o the d namic response of the structure without the influence of external constraints, which is cr cial or predicting the beha ior o components in real-world scenarios especiall in aerospace and mechanical applications

at ral re encies pla a ndamental role in determining a str cture's stiffness (with a direct relationship described b where m represents the mass o the str ct re This orm la indicates that an increase in natural frequency corresponds to higher stiffness, whereas an increase in mass yields lower stiffness. Natural frequencies indicate the rates at which the structure vibrates naturally when disturbed. Stiffness measures a structure's resistance to deformation under an applied force, influencing how the str ct re responds to e ternal loads and ibrations

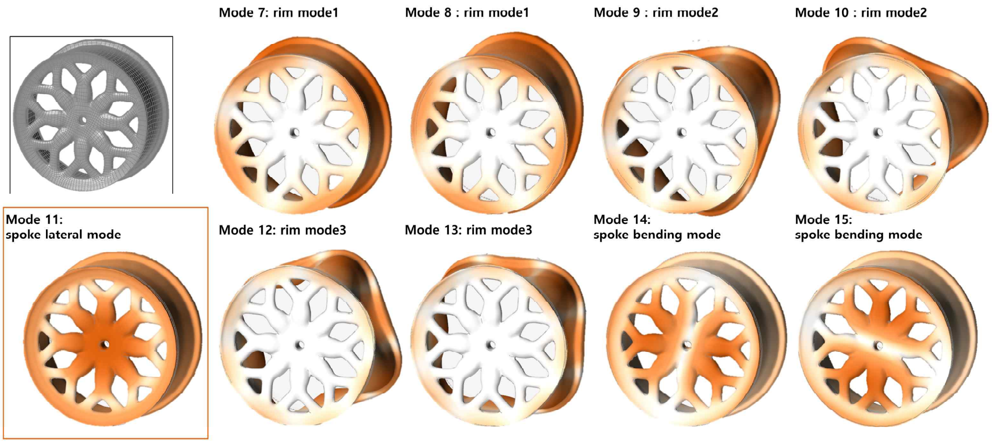
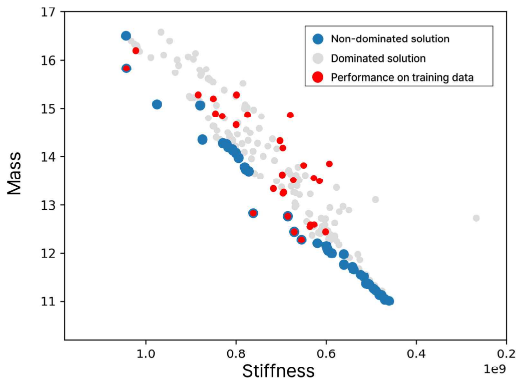


Figure 6: Modal analysis results: mode shapes across different modes

In this study, the stiffness was specifically calculated using the 11th mode, which is identified as the lateral mode of the spokes. The lateral mode in- ol es ibrations that occ r perpendic lar to the wheel s plane directl a - ecting the str ct ral integrit and per ormance o the wheel design ig

$$m \quad \pi f$$

This experiment conducted multi-objective optimization with stiffness and mass as ob ecti es The res lts presented in ig a ill strate the e ol tion o shape designs positioned along the areto ront across m ltiple generations These o tcomes demonstrate the proposed method s abilit to contin o sl e plore data within a low-dimensional latent space rather than sampling and optimizing disconnected regions o the data space The method successfully identifies meaningful candidate designs around the Pareto front, e en when compared to the training data rthermore ig b re eals that the proposed approach being ndamentall data-dri en enables realistic e ploration within a design space closel related to e isting shapes otabl the generated designs consistentl e hibit high alit irrespecti e o their origin rom either parametric or generati e design methods The e periment thus demonstrates the capability of the proposed methodology to effectively e plore and optimize within a contin o s low-dimensional data space rather than merel selecting sol tions rom isolated and discrete regions



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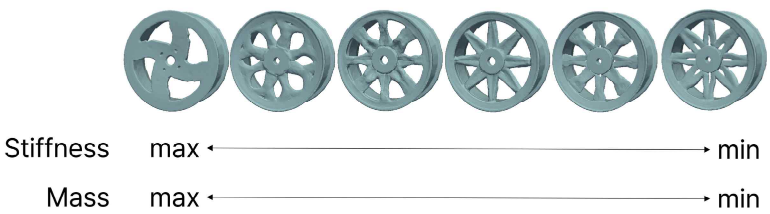


b on-dominated sol tions rom wheel samples

ig re Optimization e periment res lt sing two datasets

D-based Shape Optimization

ig re Di erse car shape e pression

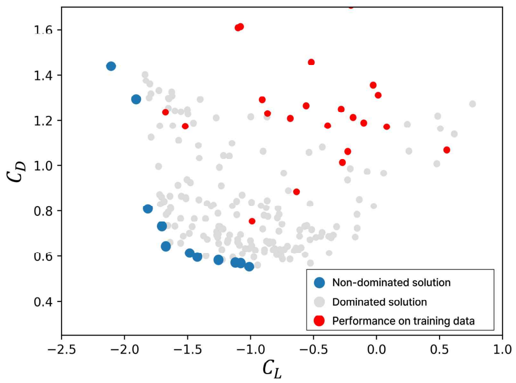
periments anal zing car aerod namics were cond cted as part o a generati e model-based shape optimization st d n this wor the Open OA sol er was set p or an incompressible e nolds A eraged a ier-Sto es anal sis sing air The sim lation sed Simple oam to capt re the drag and lift forces generated as an external airflow, moving at 10 m/s, passes by the ehicle

D car models were so rced rom the Shape et dataset a widel sed benchmar in D graphics rom tho sands o models those with clearl disting ished internal and e ternal components and s itable mesh conditions or D anal sis were selected The st d incl ded ario s car categories s ch as sedans sports cars cars with rear spoilers and sport tilit ehicles

nli e traditional approaches where e plicit parameterization is re ired to represent and compare topological changes this data-dri en methodolog le erages generati e models to directl capt re the inherent shape eat res This allows or the representation and comparison o topological ariations solel based on the inp t data enabling the optimization process to preserve fine details that conventional parameterizations often miss

sing the Open OA sim lation ramewor a generati e model was trained on a selection o Shape et car models lti-ob ecti e optimization was per ormed to minimize the $_D$ and the $_L$

The res lts are shown in ig a As the generations progressed impro ed designs emerged and grad all con erged toward the areto ront ollowing a trend similar to pre io s e periments otabl the generated candidates demonstrated s perior aerod namic per ormance compared to the training data rthermore compared with the earlier wheel dataset st d ariations in car shapes exhibited a more pronounced influence on the objective al es res lting in a broader distrib tion along the areto ront ig b rther highlights the di ersit o optimized shapes spanning ario s ehicle categories rom sports cars to sedans ill strating the model s capabilit to e plore and generate di erse areto-optimal designs nder the gi en ob ecti es



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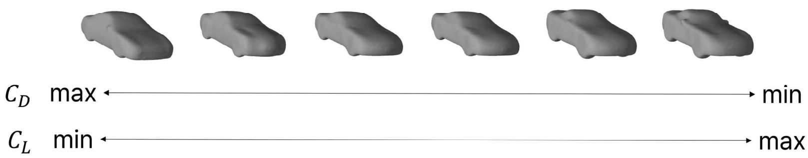

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ig re D-based car shape optimization res lts

Optimization between Two Shapes

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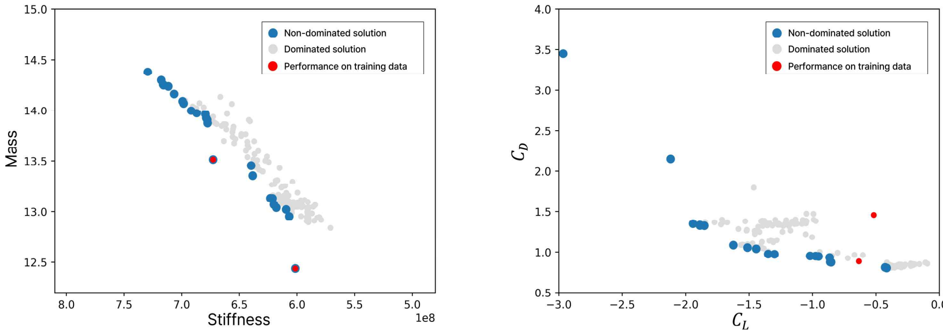
ilding on the o ndational wor described in Sections and this combined section e amines the application o the proposed optimization ramewor with a limited dataset sing onl two shapes or both wheels and cars n contrast to the pre io s e periments which emplo ed a larger and more aried dataset o designs these e periments were designed to e al ate the methodolog s rob stness and adaptabilit nder e tremel limited data a ailabilit dentical e perimental conditions to those described in Sections and were maintained to assess the optimization ramewor s ability to derive meaningful design variations, with the only differences being a red ced n mber o data and a lower-dimensional latent space The reconstr ction acc rac obtained with this two-sample set p is s mmarized in Table

As shown in igs a and b the optimization algorithm established a coherent areto ront e en rel ing on st two data points or training or the wheel shapes, the Pareto front based on stiffness and mass axes reveals that the model co ld generate di erse and engineering-rele ant ariations despite limited data n ig c the model trained on onl two wheel designs, exhibits a smooth interpolation along the Pareto front, effectively capturing design variations as if performing parametric adjustments in stiffness and mass This beha ior indicates that the A co ld sim late a continuum of structural adjustments between the limited initial designs, reflecting adaptabilit in e ploring optimal shapes within minimal data constraints Similarl in the case o car shapes the optimization process generated significant aerodynamic variations between the two baseline designs. As shown in ig d the model maps meaning l shape changes that correlate with aerodynamic performance. The algorithm effectively balanced the aerodynamic ob ecti es and ph sical design limits achie ing a range o optimized forms that reflect varying levels of aerodynamic efficiency within the specified constraints

These findings underscore the strength of the proposed methodology in handling datasets with limited di ersit while still prod cing al able and rele ant design o tp ts Altho gh the initial data di ersit is constrained the model effectively explored the potential design space, extracting patterns and structural characteristics that offer insight into performance optimization. This investigation thus affirms the adaptability of computational design methods nder conditions o sparse data showcasing that e en with minimal input, significant design insights can be obtained.



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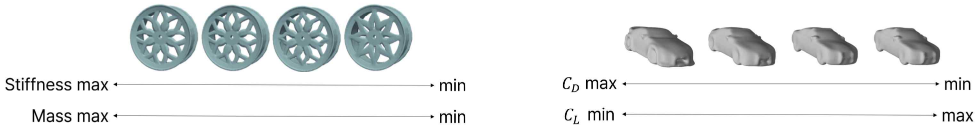

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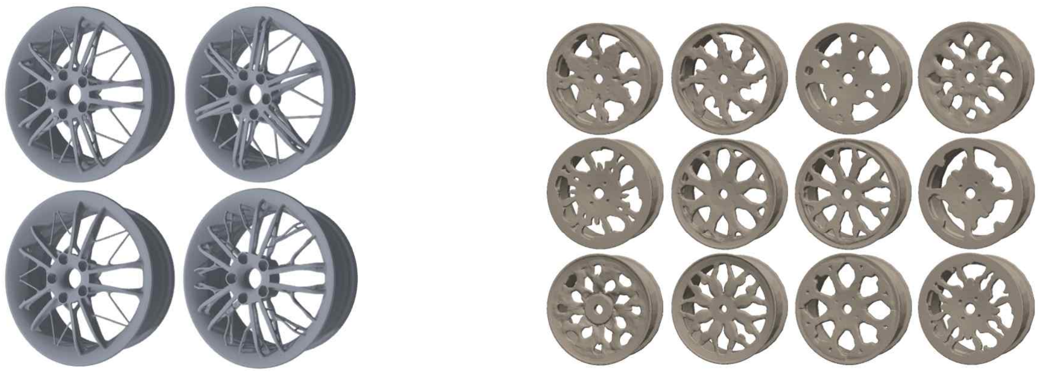
ig re perimental res lts on e tremel limited data

oncl sion

This st d proposes an engineering per ormance-based D shape optimization process that o ercomes the drawbac s o e isting methodologies while incorporating the ad antages o con entional DG -based shape optimization tensi e h per-parameter t ning across ario s architect res was cond cted to select a s itable model or the proposed approach le eraging a simple reg larization term and ad anced encoding techni es a model was de eloped that is well-s ited or the limited dataset a scenario typically considered one of the most challenging in the engineering field. Furthermore three e periments were per ormed to highlight the ad antages o this approach over conventional methods. The key findings of the study can be s mmarized as ollows

Data S nthesis Section details an e periment in which two distinct datasets are learned and optimized within the same latent space This data-driven method, which does not require the explicit definition of parameters, enables exploration into previously undefined regions, where datasets ig a and b are s nthesized ill strated in ig b



a Generati e design with topolog optimization

b Generated shapes sing the proposed method

ig re omparison o shapes between the two methodologies

- Generalizabilit ample e periments were cond cted sing wheel data or str ct ral anal sis and car data or D two o the most commonl emplo ed n merical anal sis in shape optimization Section demonstrates that the optimization process s ccess ll e plores non-dominated sol tions that o tper orm the per ormance o the act al data

- Realizable Design: The proposed framework defines shapes by learning and e tracting eat res rom real data As ill strated in ig a the data generated b topolog optimization t picall consist o a large amo nt o nstr ct red data with prono nced straight branches n contrast ig b shows shape e pressions that closel resemble those obser ed in real-world applications

The presented framework holds significant promise across two engineering applications n domains s ch as car and wheel design as emplo ed in this study, the superiority of a shape cannot be defined solely in terms of engineering performance. These areas are inherently influenced by end- ser pre erences with design iterations occ rring thro gh close collaboration between ind strial designers and engineering designers who determine the overall form [71]. In this context, the proposed framework offers the ad- antage o e ploring a data space that closel resembles real-world samples This allows it to incorporate aesthetic considerations and man act rabilit alongside antitati e engineering metrics This capabilit pa es the wa for extending the framework to collaborative efforts within industrial design.

oreo er the proposed approach can be integrated with recent ad ances in uncertainty quantification to develop task-aware surrogate models within an active learning paradigm. By defining objective values based on uncertainty rather than solel on engineering per ormance the ramewor can acilitate more effective data sampling and contribute to the continuous improvement o s rrogate model per ormance

Despite these promising res lts the proposed ramewor e hibits a notable limitation stemming rom its strong dependence on training data ith ample data, the model can effectively generate a wide variety of design candidates. However, in scenarios where available training data are significantly limited s ch as the e treme case disc ssed in Section the per ormance range o the generated sol tions becomes s bstantiall constrained omparing per ormance distrib tions rom Sections and it is e ident that the distrib tion in Section co ers onl a small design region hile increased data allows the e ploration o broader design spaces limited data imposes inherent interpolation constraints highlighting the critical importance o selecting appropriate training samples

oreo er the metrics sed to e al ate generati e models ha e limitations. Most prior studies assume that a sufficient volume of training data is available when defining their problems. Consequently, metrics such as CD, D and O are well-s ited or e al ating large datasets b t ma not be appropriate in the “small data” regime defined here. Due to the use of a antitati e shape e al ation termed engineering per ormance the broad spread o per ormance distrib tions permits the in erence that the latent space is smoothl str ct red Howe er D D and O al es alone do not g arantee that a generati e model has learned a meaning l latent representation There ore there is a need or metrics capable o e al ating whether a meaningful latent space can be defined even when only a small amo nt o data is a ailable

n this conte t a gmenting the limited dataset with strategicall meaning l data points is essential to enhance both generati e and predicti e model performance. Defining what constitutes “meaningful data” for the model and empirically validating these definitions will be discussed as part o t re research

ediT a thorship contrib tion statement

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